

THE STANDARD MODEL WITHIN NONCOMMUTATIVE GEOMETRY: A COMPARISON OF MODELS

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Abstract

Algebraic Yang-Mills-Higgs theories based on noncommutative geometry have brought forth novel extensions of gauge theories with interesting applications to phenomenology. We sketch the model of Connes and Lott, as well as variants of it, and the model developed by a Mainz-Marseille group, by comparing them in a schematic way. The role of fermion masses and mixings is discussed, and the question of possible parameter relations is briefly touched.

1. Introduction

Application of ideas of noncommutative geometry [1] to particle physics has brought forth, over the last couple of years, novel and phenomenologically interesting extensions of Yang-Mills theories [2, 3, 4, 5, 6], most of which were developed simultaneously. In fact, there is by now a problem of nomenclature because people have become used to associate noncommutative versions of the standard model with the specific framework proposed by Connes and Lott [2] although there are alternatives whose construction principles and whose results, with regard to physics, are different. One of the purposes of this lecture is to make a comparison between two of these approaches both of which have the virtue of having been elaborated in some detail. Therefore, in order to avoid that confusion I propose to call the class of extended Yang Mills theories using ideas of noncommutative geometry generically *algebraic Yang-Mills-Higgs theories*, independently of whether or not they follow Connes' specific constructive framework.

In sec. 2 I shortly describe the Connes-Lott construction, including a variant of it [7], and the model proposed by a Mainz-Marseille group, sketch the comparison between them, work out similarities and point out salient differences. Sec. 3 addresses the role of fermion masses and fermion state mixing in these models. In sec. 4 I comment on possible relations between parameters of the fermion sector that are claimed to follow from the Connes-Lott model.

2. Algebraic Yang-Mills-Higgs theories

I shall describe primarily the model proposed by Connes and Lott [2] including a variant of it that was proposed by the Leipzig group [7], and the Mainz-Marseille model but much of what will be said also applies to other constructions. Let me begin by listing the *common features*:

(i) What the models have in common, at the algebraic level, is a differential algebra, (Ω^*, d) , which may be \mathbb{N} -graded or \mathbb{Z}_2 -graded. The algebra is equipped with a product, in most cases associative, which is not the same in different models. In an appropriate limit applied to the algebra, one recovers the de Rham algebra $(\Lambda^*, d_C)_{\mathbb{N}}$, with d_C the ordinary Cartan exterior derivative, the product being the familiar wedge product. That is to say, one starts from an algebra \mathcal{A} , commutative or noncommutative, replacing $C^\infty(M)$, the algebra of smooth functions over Minkowski space M , and constructs a differential algebra $\Omega^*(\mathcal{A})$ from it, with, of course, $\Omega^0 = \mathcal{A}$.

(ii) At grade one the space Ω^1 is often the same so that the generalized connection \mathbf{A} is also the same. However, the differentials d and, hence, the generalized curvature \mathbf{F} differ.

(iii) The generalized connection \mathbf{A} incorporates the ordinary, spin-1, gauge boson fields, say V , W , etc, and the Higgs multiplet(s), say Φ .

(iv) The action (usually) exhibits spontaneous symmetry breaking, the Higgs coming about rather naturally as a geometric phenomenon. For example, in the Mainz-Marseille model there is a direct link between spontaneous symmetry breaking in the electroweak sector and maximal parity violation of charged weak interactions - a most remarkable feature.

The *differences* between different constructions, in short, may be grouped as follows.

(a) The role of the Dirac operator is rather different in different constructions. In the Connes-Lott model, following the very spirit of Connes' noncommutative geometry, the Dirac operator is a fundamental ingredient that determines the metric properties. In the Mainz-Marseille model the Dirac operator is a derived quantity. One first constructs the bosonic sector of the standard model by itself, then incorporates the fermions in chiral representations.

(b) The explicit form of the Higgs potential comes out different, at the classical level, and I will comment on this difference below.

(c) The assignment of quantum numbers of quarks and leptons needs some fixing in the Connes-Lott approach while it comes out right, without further input, in Mainz-Marseille. Obviously, this is important in the context of cancellation of anomalies.

(d) The limit of "vanishing noncommutative structure", in the two classes of models, is technically different. While it is obvious in the Mainz-Marseille model it is less transparent in Connes-Lott.

(e) The interpretation in terms of the underlying geometry, although it is not fully understood as yet, may be quite different. Connes' approach is the deeper and more ambitious one because, qualitatively speaking, it aims at replacing the geometry of spaces by the study of algebras. For this reason it lends itself rather naturally to inclusion of gravity in constructing actions - a feature foreign to the more pragmatic Mainz-Marseille model. The latter, on the other hand, has the virtue of assigning a genuine dynamic role to fermion fields of definite chirality, in accord with what the phenomenology of electroweak interactions keeps telling us.

2.1 The Connes-Lott approach

This approach is based on a spectral triple $(\mathcal{A}, \mathcal{H}, D)$, with \mathcal{A} a unital star algebra, \mathcal{H} a Hilbert space (of fermionic states), and D a generalized Dirac operator. From this triple one constructs first the universal differential envelope $(\tilde{\Omega}^*(\mathcal{A}), \tilde{d})$, and, given a representation of the algebra on \mathcal{H} , a representation of that universal object on the Hilbert space by means of bounded linear operators, viz.

$$\begin{aligned} \pi &: \tilde{\Omega}^*(\mathcal{A}) \longrightarrow L(\mathcal{H}) \\ a_0 \tilde{d}a_1 \dots \tilde{d}a_n &\longmapsto \underline{a}_0 [D, \underline{a}_1] \dots [D, \underline{a}_n] . \end{aligned} \tag{1}$$

(The representation of the element a_i of the algebra on \mathcal{H} is denoted \underline{a}_i .) For

the construction of the standard model the Dirac operator, very schematically, will have the form $D = i\gamma^\mu \partial_\mu + D_M$, the discrete piece being

$$D_M = \left(\begin{array}{cc|cc} \mathbf{0} & M & \mathbf{0} & \mathbf{0} \\ M^\dagger & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & M^* \\ \mathbf{0} & \mathbf{0} & M^T & \mathbf{0} \end{array} \right), \quad (2)$$

where M is the fermionic mass matrix, i.e.

$$M = \begin{pmatrix} M_u \otimes \mathbb{1}_3 & & \\ & M_d \otimes \mathbb{1}_3 & \\ & & 0 & M_\ell \end{pmatrix}, \quad (3)$$

where M_u , M_d , and M_ℓ denote the mass matrices of *up*-type quarks, of *down*-type quarks, and of charged leptons, respectively, M_d containing the empirical Cabibbo-Kobayashi-Maskawa (CKM) mixing.

There is a technical difficulty at this point whose resolution, at the classical level, has profound physical consequences: the projection π , eq. (1), fails to respect the differential structure of $\tilde{\Omega}^*$. (There exist elements $b \in \tilde{\Omega}^*$ for which $\pi(b)$ vanishes while $\pi(\tilde{d}b)$ does not.) Therefore, one has to divide out the ideals

$$\mathcal{J}^k(\mathcal{A}) = \mathcal{K}^k + \tilde{d}\mathcal{K}^{k-1}, \quad \text{where } \mathcal{K}^k = \ker \pi \cap \tilde{\Omega}^k,$$

sometimes called the "junk", thus obtaining

$$\Omega_{DY}^k = \tilde{\Omega}^k(\mathcal{A}) / \mathcal{J}^k(\mathcal{A}) = \pi \left(\tilde{\Omega}^k(\mathcal{A}) \right) / \pi \left(\mathcal{J}^k(\mathcal{A}) \right). \quad (4)$$

I have given the index DY to this object, because the operator D , when supplemented by the gauge and Higgs fields, yields both the gauge and the Yukawa interactions and, thus, should be termed Dirac-Yukawa operator.

The exterior derivative, that belongs to this differential algebra, is defined by eq. (1), while the product, denoted here by \odot , reflects the division by the "junk".

The choice of the algebra is suggested by the gauge group(s) one wishes to obtain and, for the purposes of the standard model, is taken to have the product form

$$\mathcal{A} = \mathcal{A}_M \otimes C^\infty(M), \quad (5)$$

with \mathcal{A}_M a block-diagonal matrix algebra which contains the gauge groups as the subsets of its unitaries. In the case of the standard model one usually takes

$$\mathcal{A}_M = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}),$$

times an appropriate number of copies of \mathbb{C} in order to keep track of the number of generations. The first two terms contain the gauge group of electroweak interactions, the last term contains the gauge group of QCD.

Thus, at the algebraic level, the resulting differential algebra is

$$(\Omega_{DY}^*(\mathcal{A}), \odot, D_{DY})_{\mathbb{N}} , \quad (6)$$

with the product \odot and the differential as defined above.

Before I turn to the Mainz-Marseille approach I wish to add a further comment. The Connes-Lott construction, as sketched here, is also the one followed by [8] and by [9] as well as by others. The Leipzig group has varied the theme by using projective modules (whereby the algebras may be chosen smaller than above)[10] and/or by replacing the algebra by a Lie algebra [7]. The first choice brings the Connes-Lott construction closer to the Mainz-Marseille one, to be described next, although there is no complete equivalence. The second choice has somewhat different phenomenological consequences on which we comment in sect. 4 below.

2.2 The Mainz-Marseille model

The algebraic structure of the Mainz-Marseille construction can be summarized as follows, again very schematically: Let $M_3(\mathbb{C})$ be the set of 3×3 -matrices, \mathbb{Z}_2 -graded by means of the grading automorphism $\Gamma = \text{diag}(1, 1, -1)$. This means that with $M \in M_3(\mathbb{C})$, the projections

$$M + \Gamma M \Gamma \quad \text{and} \quad M - \Gamma M \Gamma$$

are the even and odd parts of M , respectively. The differential algebra is taken to be the skew-tensor product

$$\Omega_{MM}^*(X) = M_3(\mathbb{C}) \hat{\otimes} \Lambda^*(X) , \quad (7)$$

where $\Lambda^*(X)$ are the exterior forms on (flat) space-time X . The object (7) contains two gradings, the \mathbb{Z}_2 of the matrix factor and the \mathbb{N} -grading of exterior forms. When the latter is turned into a \mathbb{Z}_2 -grading for odd and even forms (i.e. by taking the exterior form grade modulo 2), $\Omega_{MM}^*(X)$ inherits a \mathbb{Z}_2 bi-graded structure. Its elements are form-valued matrices whose total grade is the sum, modulo 2, of the form grade (even or odd) and the matrix grade. In other terms, the product that goes with the differential algebra of this model and that we denote by \bullet here, must be consistent with that graded structure but, obviously, there is nothing analogous to the division by the ideals of eq. (4). The differential d_{MM} , finally, is chosen to be

$$d_{MM} = d_C + [\eta, \cdot]_g , \quad (8)$$

where η is a fixed, odd element of $M_3(\mathbb{C})$, and where $[\cdot, \cdot]_g$ denotes the graded commutator.

Thus, algebraically, the Mainz-Marseille model has the structure

$$(\Omega_{MM}^*(X), \bullet, d_{MM})_{\mathbb{Z}_2} . \quad (9)$$

Here again I wish to add a few remarks. The action of d_{MM} on an element M of $M_3(\mathbb{C})$ is defined in an essentially unique way [6]. Indeed, one shows that the apparent freedom in choosing the odd element η is a freedom of choosing a phase and a "strength". The former corresponds to the freedom of choosing the vacuum point in the degenerate Higgs potential, whereby all choices are physically equivalent, the latter can always be absorbed in the (only) mass scale that appears in the connection form. Furthermore, one sees that the bi-graded structure described above effectively means embedding the Lie algebra of $SU(2)_L \times U(1)$ of electroweak interactions in the minimal graded Lie algebra $su(2|1)$ [4, 5]. Thus, in the actual calculations one may either use the explicit rules defined above, or the algebra structure encoded in the graded commutator of $su(2|1)$. The two methods are completely equivalent. Note, however, that this graded Lie algebra neither represents a new symmetry of the theory nor is any attempt made to gauge it. I mention this because there was some confusion with the much earlier work of Ne'eman, Thierry-Mieg, Fairlie and others (see, e.g. [15] and further references in [4]) who also used $su(2|1)$ in the context of electroweak interactions. A closer examination of their pioneering work shows that the role of this algebra is very different here.

2.3 A toy model for comparison

It is instructive to illustrate and to compare the two approaches by a simple example [11] which does show all relevant features but avoids the complexity of the full standard model. For that purpose choose the matrix algebra in the Connes-Lott approach to be $\mathcal{A}_M = \mathbb{C} \oplus \mathbb{C}$, and, in the Mainz-Marseille case, to be $M_2(\mathbb{C})$, equipped with the grading automorphism $\Gamma = \text{diag}(1, -1)$. The odd element η in eq. (8) here is chosen to be

$$\eta = i \begin{pmatrix} 0 & c \\ \bar{c} & 0 \end{pmatrix},$$

with c a dimensionless complex number, while the Dirac-Yukawa operator in the corresponding Connes-Lott construction is $D = \mu\eta$, with μ a constant with dimension mass. In constructing the differential algebra $\Omega_{DY}^*(\mathcal{A})$ it is not difficult to see that the "junk", for the first three grades, is given by

$$\pi(\mathcal{J}^0) = \{0\} = \pi(\mathcal{J}^1), \quad \pi(\mathcal{J}^2) = M_0 \otimes \Lambda^0(X), \quad (10)$$

with M_0 an even matrix, $\Lambda^0(X)$ functions on X . Furthermore, the differential d_{DY} can be written in a form analogous to (8). In essence, this means that

$$\Omega_{DY}^2(\mathcal{A}_M \otimes C^\infty(X)) \cong \begin{pmatrix} \Lambda^2 & 0 \\ 0 & \Lambda^2 \end{pmatrix} + \begin{pmatrix} 0 & \Lambda^1 \\ \Lambda^1 & 0 \end{pmatrix},$$

and that in calculating products \odot and differentials d_{DY} , all terms of the form

$$\begin{pmatrix} \Lambda^0 & 0 \\ 0 & \Lambda^0 \end{pmatrix}$$

are dropped. Note that this truncation does not happen in the Mainz-Marseille case.

In either model the generalized connection is the same (at grades 0 and 1 the spaces Ω^k coincide). It reads, in appropriate units,

$$\mathbf{A} = i \begin{pmatrix} V & \Phi \\ \Phi^* & W \end{pmatrix}. \quad (11)$$

At grade 2, however, the models differ, for the reasons explained above. In the Connes-Lott case the generalized curvature of the toy model is

$$\begin{aligned} \mathbf{F}^{(CL)} &= d_{DY} \mathbf{A} + \mathbf{A} \odot \mathbf{A} \\ &= i \begin{pmatrix} d_C V & -d_C \Phi - i(V - W)(\Phi + 1) \\ -d_C \Phi^* + i(V - W)(\Phi^* + 1) & d_C W \end{pmatrix}. \end{aligned} \quad (12)$$

Obviously, in calculating the Lagrangian from (12) one will find

$$\mathcal{L}^{(CL)} = -\frac{1}{4} F^{(V)2} - \frac{1}{4} F^{(W)2} + 2 (\mathbf{D}\Phi)^* (\mathbf{D}\Phi), \quad (13)$$

i.e. an expression that does contain the correct covariant derivative of the Higgs field

$$\mathbf{D}\Phi = d_C \Phi + i(V\Phi - \Phi W) + i(V - W),$$

but no Higgs potential.

The same calculation for the Mainz-Marseille case gives a different result for the curvature, viz.

$$\begin{aligned} \mathbf{F}^{(MM)} &= d_{MM} \mathbf{A} + \mathbf{A} \bullet \mathbf{A} \\ &= i \begin{pmatrix} d_C V - (\Phi + \Phi^* + \Phi\Phi^*) & -d_C \Phi - i(V - W)(\Phi + 1) \\ -d_C \Phi^* + i(V - W)(\Phi^* + 1) & d_C W - (\Phi + \Phi^* + \Phi\Phi^*) \end{pmatrix}. \end{aligned} \quad (14)$$

The Lagrangian obtained from this expression reads

$$\mathcal{L}^{(MM)} = \mathcal{L}^{(CL)} + U(\Phi), \text{ with} \quad (15)$$

$$U(\Phi) = 2(\Phi + \Phi^* + \Phi\Phi^*)^2. \quad (16)$$

Note that it exhibits spontaneous symmetry breaking, the potential $U(\Phi)$ stemming from the new, derivative-free terms in the diagonal of \mathbf{F} . (The Higgs field appears here in a dimensionless form. The physical field is related to it by a dimensionfull scale factor.) While the gauge field ' γ ' = $V + W$ remains massless, the field ' Z ' = $V - W$ becomes massive, the mass term stemming from the off-diagonal terms in \mathbf{F} . The original gauge symmetry $U(1) \times U(1)$ is broken to the residual symmetry $U(1)_\gamma$.

The occurence of the Higgs potential (16) is very natural in the context of these models. Indeed, in the case of the Mainz-Marseille model spontaneous

symmetry breaking (SSB) is a direct consequence of the noncommutative differential structure encoded in (8): If one replaces η by $\rho \cdot \eta$ where $\rho \in [0, 1]$ is a parameter that is introduced temporarily to control the "amount of non-commutativity" in the model, one sees that for $\rho = 0$ there is no SSB, both gauge fields remain massless, and the model keeps its full gauge symmetry. The reason for the absence of SSB in the Connes-Lott construction of the toy model is to be found in the subtraction of the "junk". In this example taking out the ideal $\pi(\mathcal{J}^2)$, eq. (10), is equivalent to dropping the term $(\Phi + \Phi^* + \Phi\Phi^*)$ in the diagonal of (12).

2.4 The standard model case

The Mainz-Marseille construction of the electroweak sector of the standard model is fairly straightforward. As is was described elsewhere in quite some detail [3, 4, 5, 6] we just summarize the results here. The generalized connection has the form

$$\mathbf{A} = i \left(\frac{(a/2)\boldsymbol{\tau} \cdot \mathbf{W} + (b/2)\mathbb{1}_2 W^{(8)}}{(c/\mu)\Phi} \middle| \frac{(c/\mu)\overline{\Phi}}{bW^{(8)}} \right), \quad (17)$$

where $\Phi = (\Phi^{(0)}, \Phi^{(+)})^T$ now is the Higgs doublet, while the assignment of the gauge fields is obvious. The resulting Lagrangian as obtained by means of eqs. (7) - (9) is precisely the one of the standard model with SSB, the neutral Higgs field appearing in the correct, shifted, phase [6].

The Higgs phenomenon obtains a simple and transparent interpretation: A closer examination of the gauge transformations acting on \mathbf{A} shows that there is one constant connection which is invariant under all global gauge transformations. The corresponding generalized curvature is found to be

$$\mathbf{F}_{bg}^{(MM)} = i (I_3 + Y/2) \equiv Q_{e.m.}. \quad (18)$$

Thus, the model is characterized by a constant *background field*, in the internal symmetry space, which is nothing but the charge operator. This explains at once why the residual symmetry is the $U(1)$ of electromagnetism.

The analogous Connes-Lott construction also leads to the standard model Lagrangian (including QCD) with the electroweak sector in the spontaneously broken phase. There are important differences, however. The most pronounced difference occurs in the Higgs potential $U(\Phi)$ because the two parameters that determine $U(\Phi)$ are functions of the empirical fermionic mass and mixing matrix (2). In particular, as a direct consequence of subtracting the "junk", cf. eq. (4), the potential is proportional to the square of the perpendicular part of MM^\dagger ,

$$\text{tr} \left(MM^\dagger \right)_\perp^2, \quad (19)$$

where the perpendicular part of MM^\dagger is defined as follows

$$\left(MM^\dagger \right)_\perp := MM^\dagger - \mathbb{1}_n \frac{1}{n} \text{tr} \left(MM^\dagger \right),$$

with $n = \dim M$. Obviously, the trace (19) vanishes if and only if (MM^\dagger) is proportional to the unit matrix. In the case of the standard model, this happens either if there is only one generation, or if, in each charge sector, the generations are degenerate in mass. Thus, at the classical level, there is SSB only if there is more than one generation. The parameters of the electroweak sector are functions of the empirical fermion masses - in marked contrast to the Mainz-Marseille model whose bosonic sector is independent of the fermions in the theory.

3. Role of fermion masses and mixing

In the Connes-Lott approach the fermionic mass and mixing matrices (3) are taken as the essential *input* of the Dirac-Yukawa operator. This is both the strength and the weakness of the model: The empirical values of the fermion masses and of their mixing matrix elements, as determined from experiment, fix to a large extent the bosonic sector of the standard model. Of course, with these parameters interpreted as being fundamental there will be no possibility for ever calculating them or relating them to each other or to other parameters of the theory.

This is not so in the Mainz-Marseille construction. In this model the bosonic sector is obtained from the differential algebra (9) and, hence, lives on its own, independently of whether or not there are fermions in the theory. Furthermore, the model to some extent allows to relate masses and mixings in a physically plausible way [4, 14]. This is due to the specific algebraic structure of the model. Indeed, and as we mentioned above, the differential algebra (9) carries the bi-graded structure of $su(2|1)$ which, therefore, pops up as a kind of classifying algebra for the bosonic sector. It then seems natural to classify also the fermions by means of representations of $su(2|1)$. The consequences of this hypothesis are interesting and I briefly sketch them here.

It was well known from the work of Ne'eman and Thiery-Mieg [15] that quarks and leptons fit perfectly into the fundamental representations of $su(2|1)$. For example, for one generation of leptons and of quarks, respectively, we have

$$\rho^{(\ell)} = [I_0 = \frac{1}{2}, Y_0 = -1] \longrightarrow (\frac{1}{2}, -1) \oplus (0, -2), \quad (20)$$

$$\rho^{(q)} = [I_0 = \frac{1}{2}, Y_0 = \frac{1}{3}] \longrightarrow (\frac{1}{2}, \frac{1}{3}) \oplus (0, \frac{4}{3}) \oplus (0, -\frac{2}{3}), \quad (21)$$

where the right-hand side gives the decomposition in terms of multiplets (I, Y) of $SU(2) \times U(1)$. Also, a singlet, right-handed neutrino would be described by the trivial representation

$$\rho_0^{(\ell)} = [I_0 = 0, Y_0 = 0] \longrightarrow (I = 0, Y = 0) \quad (22)$$

What is new is the observation that several generations of quarks may be classified by means of reducible but indecomposable representations of $su(2|1)$, viz.

$$\rho^{(q)} \oplus \rho^{(q)} \quad \text{or} \quad \rho^{(q)} \oplus \rho^{(q)} \oplus \rho^{(q)},$$

where $\rho^{(q)}$ is given in eq. (21). These representations which offer a natural place for generation mixing, when supplemented by a natural physical assumption, lead to interesting relations between quark masses and CKM matrix elements.

Likewise, for leptons and massive neutrinos one would use

$$\begin{aligned} & \left(\rho^{(\ell)} \oplus \rho_0^{(\ell)} \right) \oplus \left(\rho^{(\ell)} \oplus \rho_0^{(\ell)} \right) \quad \text{or} \\ & \left(\rho^{(\ell)} \oplus \rho_0^{(\ell)} \right) \oplus \left(\rho^{(\ell)} \oplus \rho_0^{(\ell)} \right) \oplus \left(\rho^{(\ell)} \oplus \rho_0^{(\ell)} \right) \end{aligned}$$

so as to obtain relations between lepton masses and neutrino mixing matrices [4, 14]. Without going into the details here let me just state the additional, physical assumption and quote a typical result. The assumption is that in the absence of electroweak interactions the masses in each charge sector are equal. In the case of *two* generations this gives a parameter-free relation, to wit

(a) For quarks,

$$\cos \theta_C = \frac{\sqrt{m_u m_d} + \sqrt{m_c m_s}}{\sqrt{(m_u + m_c)(m_d + m_s)}};$$

(b) For leptons,

$$\cos \theta = \frac{\sqrt{m_1 m_e} + \sqrt{m_2 m_\mu}}{\sqrt{(m_1 + m_2)(m_e + m_\mu)}},$$

(m_1 and m_2 being the masses of the neutrinos).

In the case of *three* generations of quarks we find a pattern of the mass matrices for *up*- and *down*-type quarks, as well as for the CKM mixing matrix which is very similar to purely phenomenological analyses of CKM mixing (for references see [14]). This is the more remarkable as the phenomenological approaches start from a physical assumption which is just the opposite of ours: they suppose that initially only one generation is massive and, in fact, very heavy, the other two are essentially massless, the masses of light quarks being generated by mixing matrix elements. The analysis within the Mainz-Marseille model not only reproduces the correct mixing pattern, it also yields analytic expressions for the CKM matrix in terms of quark masses and a few parameters which are useful for further studies.

An analysis of leptonic mass matrices along the same lines is in progress [16]. The results we have obtained so far look promising and are compatible with

some of the possible evidence for nonzero neutrino masses and mixing.

4. Possible parameter relations and some conclusions

At the classical level, the Connes-Lott construction of the standard model leads to relations between m_H , the mass of the Higgs particle, and m_W , the mass of the W , in the form of rather tight inequalities and via the fermionic mass matrix (3) which, of course, is dominated by the top quark mass. These relations were studied in detail and were especially advocated by one Marseille group [8]. In a recent paper Chamseddine and Connes [17] these relations are interpreted as being valid at a grand unification scale and are continued by renormalization group equations to the low-energy regime. Wulkenhaar, finally, who makes use of a Connes-Lott framework restricting the algebra \mathcal{A}_M to be a Lie algebra, obtains somewhat different, but comparable relations [7].

These relations obviously need some comment. A first criticism that is often put forward concerns quantization. According to our present understanding the construction yields a *classical* Lagrangian which is the one of the standard model (or is very close to it) and which, in a second step, has to be quantized following the usual procedures. Then, as there is no new symmetry in the model, there does not seem to be any obvious mechanism that would protect classical, tree-level relations after quantization.

The second criticism involves a more technical point: Obviously, in deriving the action a technically rather complicated scalar product is involved. Let me illustrate this point by the example of the Mainz-Marseille model where matters are more transparent in this respect. The calculation of the Lagrangian from the square of the curvature leads to traces (over matrices and over Lie algebras) as well as to scalar products of exterior forms of grade 0, 1, and 2. As is well known the latter have no canonical normalization and, therefore, the terms in the Lagrangian which stem from these scalar products will have arbitrary relative strengths. Another way of saying this is that there will be no canonical relation between, say, m_H and m_W . If, in turn, one decided to adopt the simplest and most natural choice for these scalar products, one would fix that mass ratio, obtaining e.g. $m_H = \sqrt{2}m_W$ [4]. In the framework of Connes and Lott some authors claim having chosen the most general scalar product compatible with the requirements imposed on the theory. This is presently being investigated by Paschke et al. [18] who study more carefully these requirements and who question whether the most general scalar product has really been used. The finding is that with a more general product still compatible with the postulates of the theory, there is essentially no restriction on m_W , while m_H has an upper bound in terms of m_t , the top mass (but no nontrivial lower bound), which is numerically similar to the results of [8]. This would imply, in practice, that it would not be possible to fix the Higgs mass in terms of either m_t or m_W , at the classical level. Clearly, this problem needs further investigation.

Let me summarize by the following comments and conclusions.

Connes' construction of noncommutative manifolds is mathematically profound and physically more ambitious than other models such as Mainz-Marseille. By replacing the study of spaces by the study of algebras it enables one to investigate more general, "noncommutative" Riemannian manifolds [19]. In particular, gravity is included in an interesting way. Its application to the eminently successful standard model of strong, electromagnetic and weak forces, on the other hand, still has some problems to be solved some of which may be of technical nature, while others might need more thinking: the fixing of the $U(1)$ quantum numbers of quarks and leptons is not natural and somewhat murky. Yet, this issue is important not only for obtaining the correct couplings but also for the discussion of chiral anomalies. The theory is formulated in Euclidean space-time and there are some problems in continuing it to Minkowski signature. There is a problem with fermion doubling whereby unphysical degrees of freedom are introduced, similarly to what happens in lattice gauge theories, [20]. Finally, there is the issue of possible parameter relations that we mentioned above.

The Mainz-Marseille construction is much simpler but also less ambitious. It yields the correct Lagrangian of the standard model, without the potential freedom of choices in ordinary Yang-Mills theories. It has no problems with the $U(1)$ quantum numbers because these are fixed correctly by the classification of quarks and leptons. As a consequence, anomalies are absent from the start [21]. Unless additional ad hoc assumptions are introduced the model does not predict classical parameter relations. On the other hand it provides an attractive framework for fermionic state mixing in terms of mass matrices, with remarkable cross-relationships to phenomenology. Finally, in this approach, the geometric nature of the Higgs phenomenon is found to be compelling and is particularly transparent.

More generally speaking, algebraic Yang-Mills-Higgs theories, independently of which avenue one follows, provide more constraints and allow for less freedom in constructing gauge theories. The constraints could fail when compared to experiment and, therefore, render these theories vulnerable. It so happens that the constraints and the additional, noncommutative structure agree with our empirical information on the fundamental interactions.

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